# Features in the primordial spectra from effective theory viewpoint

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Based on JG and M. Yamaguchi, to appear

# Outline

# Why inflation?

Inflation can provide otherwise finely tuned initial conditions

| Hot big bang | Inflation |
| --- | --- |
| Horizon problem | Single causal patch |
| Flatness problem | Locally flat |
| Monopole problem | Diluted away |
| Initial perturbations | Quantum fluctuations |

Predictions of inflation are consistent with observations, but...

# Why bothering about extra structure?

Observations seem to prefer long enough slow-roll inflation

$$\mathscr{L}_{\text{eff}}[\phi] = \underbrace{\mathscr{L}_0[\phi]}_{\text{slow-roll}}$$

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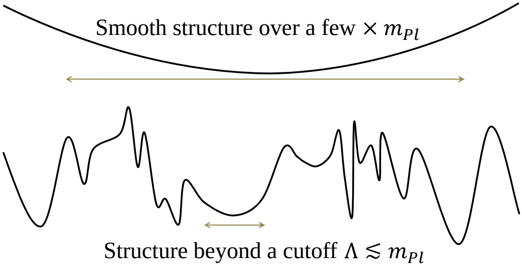
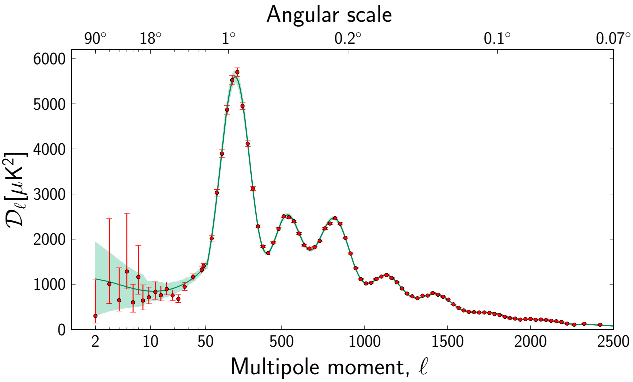
$$\mathscr{L}_{\text{eff}}[\phi] = \underbrace{\mathscr{L}_0[\phi]}_{\text{slow-roll}} + \underbrace{\sum_n c_n \frac{\mathscr{O}_n[\phi]}{\Lambda^{n-4}}}_{\text{EFT allows these}}$$




EFT introduces sub-cutoff structure: tension with observations

# Why features in the primordial spectrum?

- Intervening structure gives signals with significant deviation
- Tantalizing observational hints: have we already seen?



# Why correlated features in the primordial spectra?

- Effects of deviations permeate the whole inflationary system
- All correlation functions are correlated
- New observational handle

Q: How features are correlated in correlation functions in the effective theory viewpoint?

# Action for the Goldstone mode

In the decoupling limit with $\pi = -\mathscr{R}/H$ (Cheung et al. 2008)

$$S_\pi = \int d^4x\sqrt{-g}\left\{\frac{m_{\rm Pl}^2}{2}R - m_{\rm Pl}^2\dot{H}\left[\dot\pi^2 - \frac{(\nabla\pi)^2}{a^2}\right]\right. \longrightarrow \text{usual SR}$$

$$\left. +2M_2^4\left[\dot\pi^2 + \dot\pi^3 - \dot\pi\frac{(\nabla\pi)^2}{a^2}\right] - \frac{4}{3}M_3^4\dot\pi^3 + \cdots\right\} \longrightarrow \text{departure}$$

- $M_n^4$ is in principle independent from each other
- $M_2^4$ common to $S_2$ and $S_3$: 2- & 3-pt fct are explicitly correlated

  (cf. Achucarro et al. 2013, JG, Schalm & Shiu 2014)
- $M_3^4$ is on general argument $\mathcal{O}\left[\left(M_2^4\right)^2\right]$ so we neglect it

  (Achucarro et al. 2012)

# Features in the power spectrum

1. Consider $M_2^4$ as a perturbation:

$$S_2 = \underbrace{S_{2,\text{free}}}_{\text{no } M_2^4} + \underbrace{\int d^4x a^3 2M_2^4(t)\dot{\pi}^2}_{\equiv S_{2,\text{int}}}$$

2. Follow the in-in formalism: with $H_{2,\text{int}} = \int d^3x a^3 \left(-2M_2^4\right)\dot{\pi}^2$

$$\Delta\left\langle \pi_{\boldsymbol{k}}\pi_{\boldsymbol{q}}(\eta)\right\rangle = i\int_{\eta_0}^{\eta} ad\eta' \left\langle 0\left|\left[H_{2,\text{int}}(\eta'), \pi_{\boldsymbol{k}}\pi_{\boldsymbol{q}}(\eta)\right]\right|0\right\rangle$$

$$\equiv (2\pi)^3\delta^{(3)}(\boldsymbol{k}+\boldsymbol{q})\frac{2\pi^2}{k^3}\Delta\mathcal{P}_\pi$$

3. $\Delta\mathcal{P}_\pi$ is given by $M_2^4$: with $\mathcal{P}_\pi = \mathcal{P}_\mathcal{R}/H^2 = \left(8\pi^2 m_{\text{Pl}}^2\epsilon\right)^{-1}$,

$$\frac{\Delta\mathcal{P}_\pi}{\mathcal{P}_\pi} = \frac{\Delta\mathcal{P}_\mathcal{R}}{\mathcal{P}_\mathcal{R}} = \frac{k}{\epsilon m_{\text{Pl}}^2 H^2}\int_{-\infty}^{0} d\eta \left(-2M_2^4\right)\sin(2k\eta)$$

# Inverting the power spectrum

1. With $\widehat{\pi}_k(-\eta) = -\widehat{\pi}_k^*(\eta)$, we can oddly extend $M_2^4$ to *define* $\widetilde{M}_2^4$ as

$$\widetilde{M}_2^4(\eta) = \begin{cases} M_2^4(\eta) & \text{if } \eta < 0 \\ -M_2^4(-\eta) & \text{if } \eta > 0 \end{cases}$$

2. This extends the time integral from $(-\infty, 0)$ to $(-\infty, \infty)$:

$$\frac{\Delta\mathscr{P}_\pi}{\mathscr{P}_\pi} = -\frac{k}{\epsilon m_{\rm Pl}^2 H^2} \int_{-\infty}^{\infty} d\eta \widetilde{M}_2^4 \sin(2k\eta)$$

3. Using $\sin(2k\eta) = \left(e^{2ik\eta} - e^{-2ik\eta}\right)/(2i)$ we can invert this relation

$$\widetilde{M}_2^4(\eta) = i\frac{\epsilon m_{\rm Pl}^2 H^2}{\pi} \int_{-\infty}^{\infty} \frac{dk}{k} \frac{\Delta\mathscr{P}_\pi}{\mathscr{P}_\pi}(k) e^{2ik\eta}$$

# Bispectrum

From

$$S_3 = \int d^4x\sqrt{-g}2M_2^4\left[\dot{\pi}^3 - \dot{\pi}\frac{(\nabla\pi)^2}{a^2}\right]$$

the standard in-in formalism calculation gives

$$B_\pi(k_1,k_2,k_3) = i\widehat{\pi}^*_{k_1}\widehat{\pi}^*_{k_2}\widehat{\pi}^*_{k_3}(0)\int_{-\infty}^{\infty} d\eta\left(-2a\widetilde{M}_2^4\right)\left[6\widehat{\pi}'_{k_1}\widehat{\pi}'_{k_2}\widehat{\pi}'_{k_3}(\eta)\right.$$
$$\left. + 2(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)\widehat{\pi}_{k_1}\widehat{\pi}_{k_2}\widehat{\pi}'_{k_3} + 2\text{ perm}\right]$$

We can replace $\widetilde{M}_2^4$ with $\Delta\mathscr{P}_\pi/\mathscr{P}_\pi$ by trading $\eta$ with a deriv w.r.t. $k$:

$$\int_{-\infty}^{\infty} d\eta\eta e^{i(2k-K)\eta} = \int_{-\infty}^{\infty} d\eta\frac{1}{2i}\frac{d}{dk}e^{i(2k-K)\eta} = \frac{\pi}{2i}\frac{d}{dk}\delta\left(k-\frac{K}{2}\right)$$

# Bispectrum correlated with power spectrum

Bispectrum is specified by power spectrum and its first 2 derivs:

$$\underbrace{B_\pi(k_1,k_2,k_3)}_{=-H^{-3}B_{\mathcal{R}}} = \frac{(2\pi)^4}{(k_1k_2k_3)^2}\underbrace{\mathcal{P}_\pi^2}_{=H^{-4}\mathcal{P}_{\mathcal{R}}^2}\frac{H}{k_1k_2k_3}$$

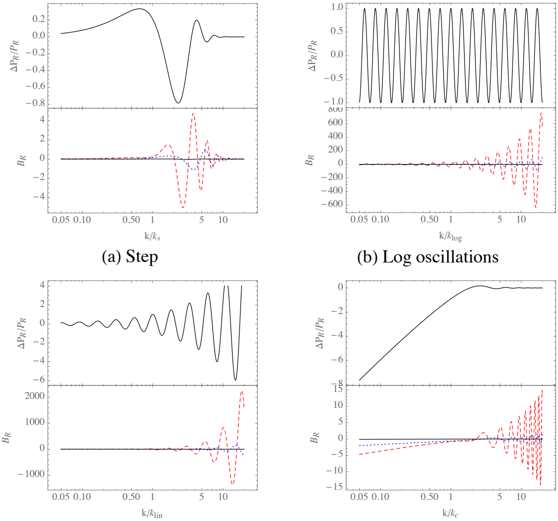
$$\times\left[A(k_1,k_2,k_3)\frac{\Delta\mathcal{P}_{\mathcal{R}}}{\mathcal{P}_{\mathcal{R}}} + B(k_1,k_2,k_3)(n_{\mathcal{R}}-1) + C(k_1,k_2,k_3)\alpha_{\mathcal{R}}\right]$$

$$A(k_1,k_2,k_3) = -\frac{1}{K^2}\sum_{i\neq j}k_i^2k_j^3 + \frac{2}{K}\sum_{i>j}k_i^2k_j^2 - \frac{1}{4}\sum_i k_i^3$$

$$B(k_1,k_2,k_3) = \frac{2}{K^2}\sum_{i\neq j}k_i^2k_j^3 - \frac{3}{K}\sum_{i>j}k_i^2k_j^2 + \frac{1}{4}\sum_{i\neq j}k_ik_j^2 - \frac{1}{4}k_1k_2k_3$$

$$C(k_1,k_2,k_3) = -\frac{1}{K^2}\sum_{i\neq j}k_i^2k_j^3 + \frac{1}{K}\sum_{i>j}k_i^2k_j^2 - \frac{1}{4}k_1k_2k_3$$

# Examples: parametrized feature models in Planck 2015




(a) Step
(b) Log oscillations
(c) Linear oscillations
(d) Cutoff

# Squeezed configuration and consistency relation

In the squeezed configuration ($k_1 \approx k_2$ and $k_3 \to 0$) $B_{\mathscr{R}} \to 0$

- $M_2^4$ first captures the speed of sound (Cheung et al. 2008a):

$$c_s^{-2} = 1 - \frac{2M_2^4}{m_{\rm Pl}^2 \dot{H}}$$

- Effects other than $c_s$ also exist but suppressed:

$$\dot{\pi} = -\frac{\dot{\mathscr{R}}}{H} \underbrace{-\epsilon\mathscr{R}}_{=+H\epsilon\pi}$$

  Cubic terms with different deriv structure: $\dot{\pi}^2\pi$ and $\pi(\nabla\pi)^2$

  (Cheung et al. 2008b)

- We need next-to-leading terms in the decoupling limit: cancellation up to $\mathcal{O}(1/c_s^2)$ and $\mathcal{O}(\epsilon/c_s^2)$ (Renaux-Petel 2010)

# Conclusions

- Features can provide information beyond slow-roll
- Features in correlation functions are all correlated
  - Leading EFT expansion coefficient $M_2^4$ sources $\Delta\mathscr{P}_\mathcal{R}$
  - We can find $B_\mathcal{R} = B_\mathcal{R}(\mathscr{P}_\mathcal{R}, n_\mathcal{R}, \alpha_\mathcal{R})$
- Different features in $\mathscr{P}_\mathcal{R}$ leads to distinctive $B_\mathcal{R}$